European Journal of Energy Research
www.ej-energy.org

Optimal Home Energy Management Considering Uncertainties in Occupancy, Consumption, and Electricity Price

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ABSTRACT

Power utilities issue demand response (DR) during the hours of peak load in order to reduce the demand on the network and provide congestion relief to overloaded circuits. While traditional residential DR programs are mainly one-way in the form of remote on/off control of air conditioning (A/C) units, residential customers can adopt a more proactive role through utilizing the capabilities of smart meters and home energy management systems (HEMS). HEMS can monitor energy rates and DR incentives, and accordingly change the temperature setpoint of the A/C unit and/or shift appliance loads from peak to off-peak hours in order to maximize financial benefits. All this can be achieved in an automated human-out-of-the-loop fashion. From the HEMS' standpoint, the task can be viewed as solving an optimization problem with the goal of reducing power consumption while maximizing financial gains. However, another equally important goal would be to ensure that the comfort level of residents, if present in the building, is not compromised. This is especially crucial during periods of extreme temperatures where maintaining an acceptable indoor temperature has a direct impact on the residents' health, especially children and the elderly. What makes this multi-objective optimization problem more challenging is the uncertain nature of some model parameters, e.g., electricity rates, building occupancy levels, and demand. This paper presents a novel solution for energy management of a smart home using DR by considering the above factors. To ensure that the solution found is feasible against all possible uncertainties, a robust model is developed and solved for a given time horizon. As shown through simulation results, considering uncertainties are necessary, since they can change the solution in a nonnegligible way.

Keywords: Demand response, demand side management, home energy management, smart home.

Published Online: May 11, 2022

ISSN: 2736-5506

DOI :10.24018/ejenergy.2022.2.3.62

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I. NOMENCLATURE

A. Indices and Sets

- s Index used for demand shiftable appliances
- q Index used for the objective functions in the multi-objective framework
- t Time index
- Q Set of objectives in the multi-objective framework
- S Set of demand shiftable appliances at the building
- T Set of time intervals

B. Parameters

- A^{F} Total window frame area of building (m^2)
- A^{W} Total transmission area of building (m^2)
- b_q Goal (target) values for objective function q
- c_t^u Nominal price of energy at time t ($/kWh)
- Δc_t^u Maximum deviation of energy price at time t compared to the nominal value ($/kWh)
- h Heat transfer constant (W/m^{2}K)
- k^{PV} PV temperature coefficient of power (°C^{-1})
- N_s Number of time periods needed for an appliance to complete its cycle (No. of hours, etc.)
- p_t Financial payment to the customer upon reducing its consumption beyond desired demand ($/kWh)
- $P_t^{d,\mathrm{des}}$ Desired demand level for the customer at time t (kW)
- $P_t^{d,(.)}$ Active power level for different sources or appliances at time t (kW), (·) could be A/C for air-conditioning, PV for rooftop photovoltaics, SH for shiftable loads, or Misc for miscellaneous loads
- $P_s^{d,\mathrm{SH}}$ Nominal active power consumption related to shiftable appliance s (kW)
- $P^{\mathrm{PV,stc}}$ Power provided by the PV panel under standard test condition (STC) (kW)
- $P^{u,\max}$ Maximum power that can be purchased from the utility at each time interval (kW)
- Q^{S}_t A/C solar load for the building at time t (kW)
- Q^{se} Total solar radiated heat flux rate (W/m^2)
- T_s^{end} Latest permissible end-time for the operation of appliance s (hour)

T_s^{start} Earliest permissible start-time for the operation of appliance s (hour)
U_t^{occ} Nominal occupancy level at the building at time t (normalized, dimensionless)
ΔU_t^{occ} Maximum deviation of occupancy at time t, compared to the nominal value (dimensionless)
α^S Solar absorptivity (dimensionless)
β_t^S Effective angle of incidence of the Sun at time t (degree)
Γ_t^c Budget of uncertainty for energy price at time t (dimensionless)
Γ_t^{Misc} Budget of uncertainty for demand of miscellaneous loads at time t (dimensionless)
Γ^{occ} Budget of uncertainty for building occupancy for the entire duration of dispatch (dimensionless)
Γ_t^{occ} Budget of uncertainty for occupancy level at time t (dimensionless)
θ_t^a Ambient temperature at time t (°C)
θ^{des} Desired indoor A/C setpoint (°C), considered as highest acceptable temperature assumed safe for residents
θ^{ref} Reference temperature (°C)
Φ_t Incident solar irradiance at PV panel at time t (W/m^2)
Φ^{STC} Incident solar irradiance at STC (W/m^2)

C. Variables

O_q Objective function q in the multi-objective optimization framework
P_t^d Total active demand for the customer at time t (kW)
P_t^{dev} Auxiliary variable used for linearizing nonlinear terms (kW)
P_t^u Total active power purchased from the utility at time t (kW)
Q_t^{Tr} A/C transmission load for the building at time t (kW)
s_q^+ Positive deficiency variable for the goal (target) of objective function q in the multi-objective setting
$u_{s,t}$ Binary variable indicating if the shiftable load s is ON at time t (= 1: ON, 0: OFF)
$w/z^{(\cdot)}$ Auxiliary variables used for developing the robust counterpart of the optimization problem. The superscript (·) can be "occ" for building occupancy, "Misc" for miscellaneous loads, or "c" for price of energy
δ Variable indicating maximum deviation from goals in the multi-objective optimization
θ_t^{set} Temperature set-point for A/C unit at time t (°C)
μ^2, μ^3 Auxiliary variables defined for developing the robust counterpart of the optimization problem

II. INTRODUCTION

Demand response (DR) is one of the pillars of the Smart Grid paradigm, where end-use consumers are encouraged to voluntarily reduce their electricity consumption levels in response to financial incentives or variable electricity rates [1]. At the residential level, DR can be deployed in the form of direct load control (DLC) or demand shifting. Through the DLC program, the utility may temporarily turn off or change the set-point of one or more appliances, usually the air-conditioning (A/C) unit, in exchange for a certain amount of credit being applied to the customer's bill. On the other hand, demand shifting is a form of interruptible DR in which certain loads may be shifted to a future time step to take advantage of more favorable electricity prices. With time-of-use (TOU) electricity rates, savvy customers can manually arrange the operation schedule of their shiftable appliances such as the washer, dryer, dishwasher, or perhaps the electric vehicle (EV) charger in order to lower their electricity bill. However, as some utilities are moving towards the more granular real-time pricing (RTP) scheme, such manual adjustments are no longer optimal (or even feasible) and instead, an automated home energy management system (HEMS) is needed that can optimize power consumption through a human-out-of-the-loop approach.

Naturally, such an automated approach can pave the way for converting residential DR into a more dynamic resource for energy management at the utility's disposal. However, it will inevitably introduce additional challenges, for instance how to ensure that indoor temperatures (when using A/C DR) do not exceed limits that might lead to health risks, how to ensure that appliance load shifting is not conducted excessively, and perhaps more importantly, how to guarantee that the load scheduling strategy devised by the HEMS is guaranteed to be optimal even when some parameters or values change. This calls for a HEMS solution that is able to simultaneously consider various objectives such as financial gains and the residents' comfort level and health, and also to safeguard against possible variations in system parameters/data with respect to their nominal values. Proposing one such system is the objective of the current paper.

The concept of automated HEMS is not new. During the past decade, many researchers have tried to address this issue by designing various solutions to optimize the power consumption of a smart home that consists of various loads. For instance, authors in [2] proposed a decision-support tool to co-optimize utilization schedule of appliances and generation schedule of distributed energy resources with the goal of maximizing net benefits. In [3], an algorithm was proposed to minimize the amount of power drawn from the utility and replace it with onsite PV power while also considering power quality issues. Authors in [4] proposed an algorithm to dynamically prioritize household appliances while taking into account the availability of onsite renewable energy and battery power. A similar method was proposed in [5] where appliances were modeled based on their start time, operation length and the acceptable delay in their operation. Other solutions have been proposed for coordinated control of home energy resources such as appliances, EV, battery and/or PV panels [6], [7]. However, all these solutions assume that system parameters and input are known with certainty. This, especially when considering the expected utilization of appliances over a certain period of time in the future, can negatively affect the applicability of the dispatch solution.

When it comes to residential load management, controlling the A/C units is a delicate matter that needs special attention. Since indoor temperature is directly related to the comfort level of residents, some authors have incorporated customer convenience into their optimization models, e.g. by defining cost of discomfort (assumed to be a linear function of temperature deviation from the desired temperature range) [8], hard constraints on upper and lower temperatures [9], [10], or heuristic models to determine thermal comfort as a function of indoor temperature, relative humidity and air motion [11], [6]. Naturally, an important aspect of A/C-centered DR solutions is to be able to predict the A/C unit consumption based on ambient temperature and other parameters. Various models have been used for this purpose, e.g. simplified state space models [6], [8], reduced-order heat transfer models [11], [12], and artificial neural networks [10].

The goal of this paper is to propose a robust solution for a DR-based home energy management system (HEMS) that can handle uncertainties in input data and parameters. The schematic diagram of the solution is illustrated in Fig. 1. At any point in time *t*, estimates for present and future building occupancy levels, future demand of miscellaneous (non-demand-responsive) loads, and future electricity rates are used to devise an optimal strategy for demand response and demand shifting. Future occupancy or demand estimates can be acquired using the historical data. However, this aspect is considered to be outside scope for the current paper, i.e., it is assumed that nominal (baseline) estimates are provided to the HEMS. We propose a multi-objective optimization framework that concurrently considers cost minimization, demand reduction, and the residents' comfort. A goal programming approach is used to ensure that no individual objective function will dominate the others. In addition, this approach allows us to guarantee the Pareto optimality of the solution. To address the uncertainty in future values of parameters such as the occupancy level or demand patterns, we employ a robust optimization model that safeguards the solution against deviations from nominal estimates. The proposed robust multi-objective (RMO) solution can be implemented in the form of a model predictive control in which the solution, moving forward in the form of a sliding window, is found at any point in time for a given time horizon.

The rest of the paper is organized as follows. Section 3 presents the formulation for the proposed multi-objective optimization problem, first with the assumption of deterministic parameters, which is then extended to cover the uncertainties in the model. The case study and simulation results are provided in section 4 of the paper. Finally, concluding remarks appear in section 5.

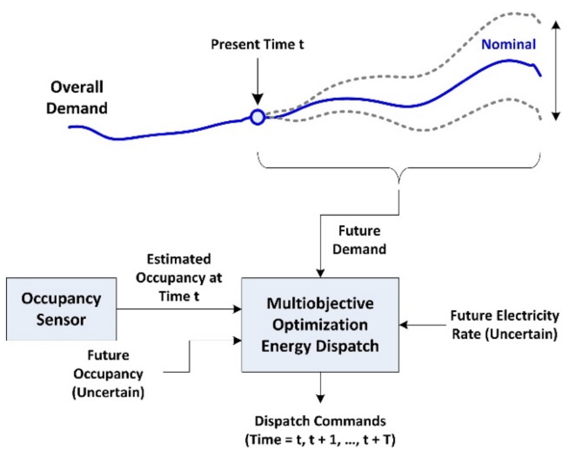

Fig. 1. Schematic diagram of the proposed solution

III. Proposed Methodology for Demand Response

A. Assumptions

It is assumed here that the customer has two options for demand response: reducing the A/C unit power consumption (by increasing its temperature setpoint) and/or shifting the demand of one or more smart appliances from peak hours to off-peak hours. Further, the DR contract between the utility and the customer is assumed to present two financial mechanisms. First, once the power utility sends a demand reduction request, the customer is expected to limit its power consumption to the level requested by the utility. Failure to do so may for instance lead to financial "cost," e.g., either penalties or disqualifying the customer from receiving financial incentives per the DR contract. Second, reduction beyond what requested by the utility may entitle the customer to additional financial reward (on top of the incentives per the DR contract). For simplicity, and without loss of generality, each time step is considered here to be 1 hour, which means that the power levels in (kW) would be equivalent in value to the energy levels (in kWh).

B. Problem Formulation

1) Objective Functions

The following objective functions are considered:

Demand Minimization: Assuming the utility has set a desired demand level of $P_t^{d,\mathrm{des}}$ for the customer at time *t*, the HEMS will try to minimize the surplus consumption beyond the target value. This objective function does not consider financial incentives and/or penalties associated with complying or failure to comply with the target demand reduction.

$$O_1 = \min \sum_{t\in \mathrm{T}} (P_t^d - P_t^{d,\mathrm{des}})_+ \tag{1}$$

where function $(\cdot)_+$ returns the argument if positive, and zero otherwise.

Cost Minimization: demand reduction must be achieved in a way that maximizes the financial gains by the customer and minimizes costs. Primarily, the customer would prefer to avoid penalties or any missed financial payments (by meeting the desired demand reduction as in (1)). In addition, whenever possible, it would prefer to reduce its consumption beyond the target amount in order to qualify for additional incentive payments. This has been expressed as in (2).

$$O_2 = \min \sum_{t\in \mathrm{T}} \{c_t^u P_t^u - p_t \cdot (P_t^{d,\mathrm{des}} - P_t^d)_+\} \tag{2}$$

A/C Temperature Control: although demand reduction by turning off the A/C or increasing its temperature setpoint is an effective way to reduce power consumption, one must ensure that indoor temperatures at the residential building are within an acceptable range in order to maintain healthy conditions for the residents, in particular children and the elderly. Hence, HEMS will try to minimize the function in (3), where the objective function is penalized by the building's occupancy level. The function is scaled by the normalized building occupancy level, so that the presence of residents assigns higher priority to it.

$$O_3 = \min \sum_{t\in \mathrm{T}} |\theta^{\mathrm{des}} - \theta_t^{\mathrm{set}}| \cdot \mathrm{U}_t^{\mathrm{occ}} \tag{3}$$

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2) *Constraints*

The above objective functions are optimized subject to the following constraints:

Power Balance: this constraint models the total power consumption at the residential unit. A simple power balance equation is used here since it provides an adequate level of accuracy for a single customer. To estimate the A/C demand, a mathematical model is used based on the work in [12]. Here, the cooling load for the A/C unit is assumed to consist of four terms, (*a*) transmission load which is the heat gain caused by the temperature difference between the building elements (e.g. walls and windows) and the outside, (*b*) infiltration load which is the heat gain due to the flow of outdoor air into the building, (*c*) solar load, i.e. heat gain due to direct solar irradiation, and (*d*) internal load which is the heat gain caused by the heat released into the building space by different equipment (such as lighting) or people. In this paper, for simplicity and without loss of generality, we only consider the first two components.

$$\forall t \in \mathrm{T}: P_t^u = P_t^d \tag{4}$$

$$\forall t \in \mathrm{T}: P_t^d = P_t^{d,\mathrm{A/C}} + \sum_{s \in \mathrm{S}} u_{s,t} P_s^{d,\mathrm{SH}} + P_t^{d,\mathrm{Misc}} - P_t^{d,\mathrm{PV}} \tag{5}$$

$$\forall t \in \mathrm{T}: P_t^{d,\mathrm{A/C}} = Q_t^{\mathrm{Tr}} + Q_t^{\mathrm{S}} \tag{6}$$

$$\forall t \in \mathrm{T}: Q_t^{\mathrm{Tr}} = h \cdot A^{\mathrm{W}} (\theta_t^a - \theta_t^{\mathrm{set}}) \tag{7}$$

$$\forall t \in \mathrm{T}: Q_t^{\mathrm{S}} = \alpha^{\mathrm{S}} Q^{\mathrm{se}} A^{\mathrm{F}} \sin \beta_t^{\mathrm{S}} \tag{8}$$

$$\forall t \in T: P_t^{d,\mathrm{PV}} = P^{\mathrm{PV,STC}} \frac{\Phi_t}{\Phi^{\mathrm{STC}}} [1 - k^{\mathrm{PV}} (\theta_t^a - \theta^{\mathrm{ref}})] \tag{9}$$

Demand Shiftable Loads: if shiftable appliance *s* is turned on at time *t* then (*a*) it must be off during the previous time steps, (*b*) it needs to remain on for the duration of time it needs to complete its cycle, and (*c*) it must turn off after the completion of the cycle. These constraints are modeled as in (10)-(11). Equation (11) ensures that demand can be shifted to a future time step only if enough time remains in the dispatch period to complete the cycle. Appliance *s* cannot start earlier than the predetermined start time and should finish its cycle prior to the predetermined end time (see (12)). Also, demand shifting is not performed partially here and if scheduled, the entire load will be shifted to a future time step.

$$\forall s \in \mathrm{S}: \sum_{t \in \mathrm{T}} u_{s,t} = N_s \tag{10}$$

$$\forall s \in \mathrm{S}, \forall t \in \mathrm{T}: u_{s,t+1} \geq \frac{u_{s,t}}{N_s} \left(N_s - \sum_{\tau=1}^{t} u_{s,\tau} \right) \tag{11}$$

$$\forall s \in \mathrm{S}: \sum_{t=1}^{\mathrm{T}_s^{\mathrm{start}}-1} u_{s,t} = 0, \quad \sum_{t=\mathrm{T}_s^{\mathrm{end}}+1}^{\mathrm{T}} u_{s,t} = 0 \tag{12}$$

A/C Temperature Limits: For health reasons, it is desired that the setpoint temperature of the A/C unit does not exceed a certain predetermined threshold. In this paper, we have considered the desired setpoint to be 23.88°C (75°F), and that the setpoint may not exceed 25.55°C (78°F) at any time.

$$\forall t \in \mathrm{T}: \left| \theta_t^{\mathrm{set}} - \theta^{\mathrm{des}} \right| \leq 1.67 \tag{13}$$

In addition, we wish to place a limit on the overall variations of temperature setpoint from the desired value. This is intended to reduce the inconvenience on the end user; otherwise, the optimization problem will force the setpoint to be at the upper limit at all times. This constraint is expressed as in (14). Moreover, to eliminate the possibility of more consumption due to overcooling of the house by the A/C unit, (15) is added.

$$\sum_{t=1}^{\mathrm{T}} | \theta^{\mathrm{des}} - \theta_t^{\mathrm{set}} | \leq 19.44 \tag{14}$$

$$\forall t \in \mathrm{T}: \theta_t^{\mathrm{set}} \geq \theta^{\mathrm{des}} \tag{15}$$

Integrality Constraints:

$$\forall s \in \mathrm{S}, \forall t \in \mathrm{T}: u_{s,t} \in \{0, 1\} \tag{16}$$

C. *Robust Counterpart*

The formulation presented in the previous section assumes that all parameters are deterministic. However, in reality this is not always the case, and there will be some level of uncertainty associated with the model. In this paper, building occupancy $\mathrm{U}_t^{\mathrm{occ}}$, demand for miscellaneous loads $\mathrm{P}_t^{d,\mathrm{Misc}}$, and electricity rates c_t^u are assumed to be uncertain (subject to variations with respect to their nominal values, which for instance could have been forecasted based on historical data). As for building occupancy, it has been assumed that the value for the first time-interval, i.e., present time, is determined using occupancy sensors or other means, and as such, is deterministic. However, the values are assumed uncertain for future time steps (see Fig. 1). To guarantee the feasibility and optimality of the solution under worst-case scenarios, the optimization problem presented in the previous section has been made robust by developing its robust counterpart according to the budgeted uncertainty approach [13]. This is less conservative than the box RC (where it assumes all uncertain parameters are at their worst-case value simultaneously), but more tractable than the ball RC (that makes assumptions about the L_2 norm of the uncertainty). In the theory of robust counterpart [13], if a vector *c* is uncertain, its uncertainty set can be written as:

$$\Psi = \left\{ \tilde{c} = c + \sum_{k \in \mathrm{K}} \zeta_k \cdot \hat{c}_k : \zeta \in \mathrm{Z} \subset \sim^{\mathrm{K}} \right\}, \tag{17}$$

where Z is the perturbation set and is assumed to be closed and convex. The term *c* in (17) is the nominal value of *c*, whereas $\hat{c}_k$ denotes the basic shifts to the nominal value (which are scaled by uncertainty parameter ζ_k belonging to Z). For the budgeted uncertainty model, Z is defined as [13]:

$$\mathrm{Z} \equiv \{ \zeta \in \sim^{L} : \|\zeta\|_\infty \leq 1, \|\zeta\|_1 \leq \gamma \} \tag{18}$$

where $\gamma \in [1, L]$ is a given uncertainty budget, with *L* indicating the number of uncertain parameters.

The first step to model the optimization problem under uncertainties is to develop the robust counterpart of all constraints that include uncertain parameters. To do this, equations (1) and (2) are first linearized using auxiliary variables. In addition, since O_2 and O_3 include uncertain parameters, they need to be written in their epigraph form in order to derive their robust counterpart. This is accomplished as follows, where μ^2 and μ^3 are auxiliary variables defined for the two objective functions:

$$\mathrm{O}_2 = \min \mu^2, \text{subject to: } \mu^2 \geq \sum_{t \in \mathrm{T}} (c_t^u P_t^u - p_t \cdot P_t^{\mathrm{dev}}) \tag{19}$$

$$\mathrm{O}_3 = \min \mu^3, \text{subject to: } \mu^3 \geq \sum_{t \in \mathrm{T}} (\theta_t^{\text{set}} - \theta^{\text{des}}) \cdot \mathrm{U}_t^{\text{occ}} \tag{20}$$

In the presence of uncertainties, it is often recommended to have a reserve margin for the generation-consumption balance. Therefore, constraints (4)-(6) are combined and rewritten as in (21):

$$\forall t \in \mathrm{T}: P_t^u \geq \sum_{s \in \mathrm{S}} u_{s,t} P_s^{d,\mathrm{SH}} + P_t^{d,\mathrm{Misc}} - P_t^{d,\mathrm{PV}} + P_t^{d,\mathrm{A/C}} \tag{21}$$

Considering Γ^{occ} to be the budget of uncertainty for building occupancy, Γ_t^{Misc} the budget of uncertainty for miscellaneous loads at each time interval t, and Γ^c to be the budget of uncertainty for energy price, and using the approach introduced in [13], the robust counterpart of (19)-(20) can be written as:

$$\mu^2 \geq \sum_{t \in \mathrm{T}} (c_t^u P_t^u - p_t \cdot P_t^{\text{dev}}) + \Gamma^c \cdot \max(|w_t^c|) + \sum_{t \in \mathrm{T}} |z_t^c| \tag{22}$$

$$\forall t: w_t^c + z_t^c = \Delta c_t^u \cdot P_t^u \tag{23}$$

$$\mu^3 \geq \sum_{t \in \mathrm{T}} (\theta_t^{\text{set}} - \theta^{\text{des}}) \cdot \mathrm{U}_t^{\text{occ}} + \Gamma^{\text{occ}} \cdot \max(|w_t^{\text{occ}}|) + \sum_{t \in \mathrm{T}} \left| z_t^{\text{occ}} \right| \tag{24}$$

$$\forall t: w_t^{\text{occ}} + z_t^{\text{occ}} = (\theta_t^{\text{set}} - \theta^{\text{des}}) \cdot \Delta U_t^{\text{occ}} \tag{25}$$

$$\begin{aligned} &\forall t: P_t^u \geq \sum_{s \in \mathrm{S}} u_{s,t} P_s^{d,\mathrm{SH}} + P_t^{d,\mathrm{Misc}} - P_t^{d,\mathrm{PV}} + P_t^{d,\mathrm{A/C}} \\ &+\Gamma_t^{\text{Misc}} \cdot |w_t^{\text{Misc}}| + |z_t^{\text{Misc}}| \end{aligned} \tag{26}$$

$$\forall t: w_t^{\text{Misc}} + z_t^{\text{Misc}} = \Delta P_t^{\text{d,Misc}} \tag{27}$$

Notice that in (24), $\mathrm{U}_t^{\text{occ}}$ denotes the nominal occupancy level at time t. By introducing some auxiliary variables, (22), (24), and (26) can be linearized. This process is very straightforward and has been removed here for brevity of presentation.

D. Solution Methodology

The problem to be solved can be described as minimizing O_1–O_3 subject to (4)-(16), (19)-(27). This is a multi-objective optimization problem, and is solved here using Chebyshev goal programming (CGP), because CGP provides a balance between goals, as opposed to other GP techniques that suffer from the incommensurability problem or require prioritizing the goal values [14]. CGP uses the Chebyshev L_∞ norm for measuring the distances of the objective functions to their corresponding targets (goals). It then tries to minimize the maximum deviation from any goal. This way, all objective functions are given equal priority and a balance is maintained between the goals without making the problem subjective. To ensure Pareto optimality, we adopt the methodology proposed in [15], where we add a small percentage of each original objective function to the CGP function (the ε value has been chosen as 0.05 in this study). This way, even if the specified goal is achieved, the problem continues to improve the solution by minimizing the overall objective function. The multi-objective problem is expressed as:

$$\min F = \delta + \sum_{q \in Q} \varepsilon \cdot O_q \tag{28}$$

Subject to:

$$\forall q \in Q: \frac{s_q^+}{b_q} \leq \delta, \;\; O_q - s_q^+ \leq b_q, \;\; s_q^+ \geq 0 \tag{29}$$

In the above equations, (28) is the multi-objective function that should be minimized. The last term in this equation, i.e., a percentage of each objective function, is intended to ensure Pareto optimality. The first constraint in (29) tries to minimize the distance of each objective function from their corresponding target (goal) values (set by the user). The second constraint in (29) indicates our desire for the individual objective functions to be less than their specified targets (goals). The deficiency variables s^+ convert these requirements into soft constraints.

IV. Case Study

A. Data

In this study, a typical single-family house located in Sacramento, CA is considered for proof-of-concept purposes. The building is equipped with central A/C. Without loss of generality, it is assumed that the residents are at the building at time t. Data for future occupancy levels and future demand has been based on the historical values according to the time of the day (with the values normalized). The ambient temperature data is based on a heat wave event that occurred in Sacramento, CA on July 23, 2006. The desired indoor temperature is chosen to be 23.88°C (75 °F). Power generated by PV units located on the roof of the house (according to the specifications shown in Table I) is illustrated in Fig. 2. Moreover, Table II shows the characteristics of the shiftable loads.

Table I. Characteristics of the PV units

No. Panels	Panel Size (kW)	P^{STC} (kW)	Φ^{STC} (W/m^2)	k^{PV} (°C^{-1})
2	200	200	1,000	0.004

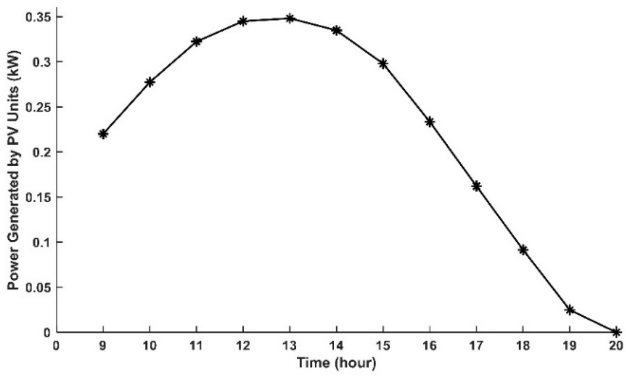

Fig. 2. Power generated by rooftop PV units based on time of day (temperature corrected).

Table II. Characteristics of Shiftable Loads

Appliance	T_s^{start}	T_s^{end}	Rated Power (kW)	N_s
S_1 (Dishwasher)	11	16	0.5	1
S_2 (Washer and Dryer)	15	18	1.5	2

B. Simulation Results

The individual goal (target) values are derived by running the deterministic optimization problem in the form of a single-objective problem (i.e., considering one objective function at a time and ignoring the rest). This provides the truly global optimum values for each objective function (an upper limit of 7.5kW for purchased power is considered when minimizing O_3 to avoid a goal value of zero). These values are then relaxed by considering a 10% margin (chosen heuristically) to determine the goals (targets) that are to be achieved in the multi-objective framework. After setting the goal values, the multi-objective optimization model is solved over a 12-hour time period, i.e., from 9:00am to 8:00pm.

For the robust model, the budget of uncertainty for the miscellaneous loads at each time interval (Γ_t^{Misc}), the overall occupancy (Γ^{occ}), and the price of electricity price (Γ^c) are set to be 0.75, 7, and 8, respectively. Naturally, to solve the deterministic case, all budgets of uncertainty are set to be zero, i.e., no deviations from the nominal values are allowed. Using the GAMS/LINDOGLOBAL solver on a desktop computer with Intel® Core™ i7-9750H and 2.60 GHz CPU and 16 GB RAM, the multi-objective deterministic and robust problems solve in 0.362 and 0.565 seconds, respectively. Optimal values for various objective functions for different models are listed in Table III.

TABLE III. OPTIMAL VALUES AND ASSIGNED GOALS

Optimal Values	Objective Functions		
	O_1 (kW)	O_2 (\$)	O_3 (°C)
SO	31.95	6.77	0.21
Goal (Target) Values	35.15	7.45	0.23
MO	35.68	7.05	0.23
RMO	37.29	12.37	0.38

SO: single objective model, MO: multi-objective deterministic model, RMO: multi-objective robust model.

Based on the results in Table III, goal values for all objective functions except for O_1 are achieved in the deterministic case. However, these goals are not met in the non-deterministic case. The impacts of uncertainties on the optimal values of the objective functions can be observed by comparing the MO and RMO cases. It can be seen that incorporating uncertainties in the problem formulation results in deteriorating the values of the objective functions O_1, O_2 and O_3 by 4.51%, 75.46%, and 65.22%, respectively. This is a further testament to the fact that it is critical for a HEMS to operate realistically by considering potential uncertainties in model and/or parameters. Failing to incorporate the uncertainties can result in violations from the desired demand and/or loss of financial incentives (or even penalties). In fact, not considering uncertainties, may cause some loads to shift to time intervals with higher electricity rates.

Fig. 3 shows the A/C temperature setpoints in both deterministic and non-deterministic cases. It can be seen that setpoints at hours 17 and 20 are lower in the non-deterministic case compared to the deterministic case. The fact that introducing uncertainties can lead to lower A/C setpoints (which is more desirable) may sound counter-intuitive, since it is natural to expect that uncertainties deteriorate all aspects of the solution. However, this phenomenon can be explained by observing the occupancy information (Fig. 4). For this particular house, average occupancy level (nominal value) is zero for hours 18, 19 and 20, which is why the deterministic model raises the temperature setpoint to the maximum value of 25.55 °C, without worsening the objective function (see objective function O_3 in (4), where setpoint deviation is weighed by the occupancy level). Conversely, in the non-deterministic case, worst-case values for occupancy are non-zero, which is why the robust multi-objective problem tries to decrease the temperature setpoints in order to keep the percentage deterioration in O_3 as low as possible.

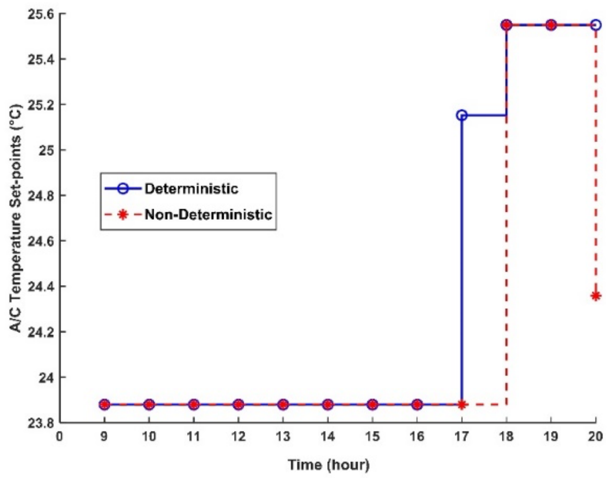

Fig. 3. A/C temperature setpoints during the dispatch period.

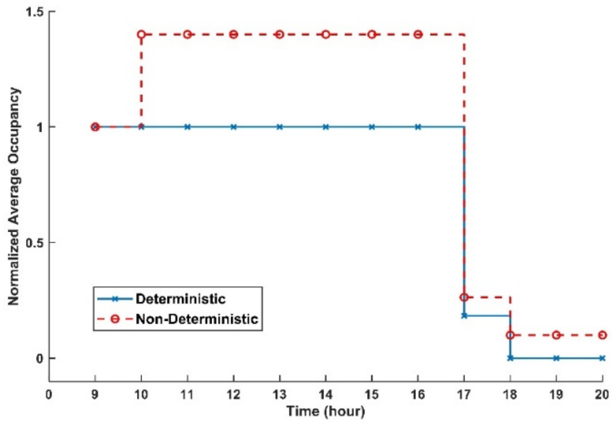

Fig. 4. Normalized occupancy level in both deterministic and nondeterministic cases. The deterministic case represents the nominal occupancy level of the building, which can be obtained by averaging the historical data.

The total power as well as A/C power consumption levels (in both deterministic and non-deterministic cases) are depicted in Fig. 5. It can be seen that the purchased power is higher for the non-deterministic case during all time intervals with the exception of hours 15 and 16. This is due to the different dispatch of washer and dryer loads in the two cases. Total purchased power in deterministic and non-deterministic cases are 72.76 kWh and 75.26 kWh, respectively. This further shows the negative effect of uncertainties in making power purchase decisions (nearly 3.5% in this example). Similarly, A/C power consumption is higher in the non-deterministic case (68.48 kWh) compared to the deterministic case (67.44 kWh) because of lower A/C temperature setpoints in the former, as explained above. This is expected since the (nearly) worst-case realization of uncertainties is considered for obtaining the optimal decisions in the non-deterministic case.

In Table IV, the dispatch status of shiftable loads is compared between deterministic and non-deterministic cases. It can be viewed that the dishwasher is dispatched the same way but washing machine and the dryer are dispatched in different hours. This has happened due to lower price volatility at hours 17 and 18. This result shows that introduction of the uncertainties in the problem forces HEMS to make different dispatch decisions for the shiftable loads.

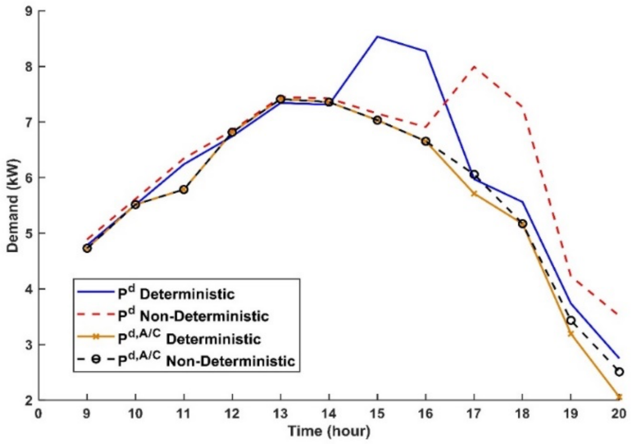

Fig. 5. Total power and A/C power consumption for the dispatch period.

TABLE IV. DISPATCH DECISIONS (1: ON, 0: OFF) FOR SHIFTABLE LOADS

Hour	Appliance			
	Dishwasher		Washer and Dryer	
	Deterministic	Non-Deterministic	Deterministic	Non-Deterministic
9	0	0	0	0
10	0	0	0	0
11	1	1	0	0
12	0	0	0	0
13	0	0	0	0
14	0	0	0	0
15	0	0	1	0
16	0	0	1	0
17	0	0	0	1
18	0	0	0	1
19	0	0	0	0
20	0	0	0	0

Finally, to see the effects of various uncertainties on the dispatch solution, a sensitivity analysis is conducted. First, parameter uncertainties are considered one at a time as well as combinations of two parameters at a time. For each combination, the problem is solved, and the results are compared in Table V. It can be seen that, compared to other parameters, uncertainties in the price of energy can have a more significant impact on the overall dispatch, which further underlines the importance of a more accurate forecast for the energy price. To further observe the effects of energy price uncertainties on the problem, the optimization model is solved using various uncertainty intervals and the changes in the three objective function optima are illustrated in Fig. 6. For a clearer presentation, the optimal values of objective functions are normalized based on their deterministic optima. The results indicate that uncertainties in energy price have the least effect on O_1 and the largest effect on O_2. Interestingly, higher levels of uncertainty in energy price will result in lower values for O_1 since price increase forces the HEMS to reduce the demand (which as a result would worsen both O_2 and O_3). Of course, it should be noted that these findings are applicable to the current case study with the given numerical data. Changes to the problem setting or the input data can affect the conclusions discussed above.

TABLE V. SENSITIVITY ANALYSIS RESULTS

Case	Optimal Values of Objective Functions					
	O_1 (kW)	Δ_1 (%)*	O_2 ($)	Δ_2 (%)*	O_3 (°C)	Δ_3 (%)*
RMO (U_t^{occ} uncertain)	35.95	0.84	7.39	4.82	0.24	4.34
RMO ($P_t^{d,Misc}$ uncertain)	36.93	3.50	7.39	4.82	0.24	4.34
RMO (c_t^u uncertain)	35.57	–0.3	11.69	65.81	0.31	34.78
RMO (U_t^{occ} and $P_t^{d,Misc}$ uncertain)	37.21	4.28	7.64	8.36	0.24	4.34
RMO (U_t^{occ} and c_t^u uncertain)	35.99	0.86	11.93	69.21	0.37	60.86
RMO ($P_t^{d,Misc}$ and c_t^u uncertain)	36.84	3.25	12.15	72.34	0.31	34.78

*Δ indicates the percentage deviation of each objective function with respect to the deterministic problem, i.e., no uncertainties.

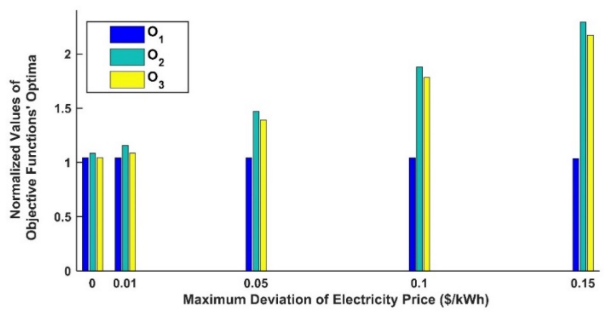

Fig. 6. Normalized values of the objective functions' optima for different values of electricity price uncertainties (i.e., deviations from the nominal price).

V. CONCLUDING REMARKS

A solution was proposed in this paper for home energy management using demand response. The problem was formulated as multi-objective robust optimization in order to allow for simultaneous consideration of different objective functions as well as inclusion of parameter uncertainties in the model. Building occupancy estimates were used in the problem formulation to ensure that control of A/C temperature setpoints is conducted in a way that negative health impacts on the residents who might be present in the building are prevented. Simulation results were provided to show how appliance scheduling (demand shifting) and A/C setpoint adjustment may change when considering uncertainties in demand, energy cost, or building occupancy. While this may not cause issues for a power utility when only considering a single customer, it can lead to significant mismatches when extended to a large number of customers.

FUNDING

This work was supported in part by the National Science Foundation under Grant 1610701.

CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

REFERENCES

[1] Mohagheghi S, Stoupis J, Wang Z, Li Z, Kazemzadeh H. Demand Response Architecture – Integration into the Distribution Management System. In Proc. *SmartGridComm*, 2010: 501–506.
[2] Pedrasa MAA, Spooner TD, MacGill IF. Coordinated Scheduling of Residential Distributed Energy Resources to Optimize Smart Home Energy Services. *IEEE Trans. Smart Grid*. 2010; 1(2): 134-143.
[3] Rajalingam S, Malathi V. HEM Algorithm Based Smart Controller for Home Power Managmenet System. *Energy and Buildings*. 2016; 131: 184-192.
[4] Byun J, Hong I, Park S. Intelligent Cloud Home Energy Management System Using Household Appliance Priority Based Scheduling Based on Prediction of Renewable Energy Capability. *IEEE Trans. Consumer Electronics*. 2012; 58(4): 1194-1201.
[5] Teng R, Yamazaki T. Load Profile-Based Coordination of Appliances in a Smart Home. *IEEE Trans. Consumer Electronics*. 2019; 65(1): 38-46.
[6] Luo F, Ranzi G, Wan C, Xu Z, Dong Z. Y. A Multistage Home Energy Management System with Residential Photovoltaic Penetration. *IEEE Trans. Industrial Informatics*. 2019; 15(1): 116-126.
[7] Wang F, Zhou L, Ren H, Liu X, Talari S, Shafie-Khah M, Catalão JPS. Multi-Objective Optimization Model of Source-Load-Storage Synergetic Dispatch for a Building Energy Management System Based on TOU Price Demand Response. *IEEE Trans. Industry Applications*. 2018; 54(2): 1017-1028.
[8] Molina D, Lu C, Sherman V, Harley R. G. Model Predictive and Genetic Algorithm-Based Optimization of Residential Temperature Control in the Presence of Time-Varying Electricity Prices. *IEEE Trans. Industry Applications*. 2013; 49(3): 1137-1145.
[9] Jo H-C, Kim S, Joo S-K. Smart Heating and Air Conditioning Scheduling Method Incorporating Customer Convenience for Home Energy Management System. *IEEE Trans. Consumer Electronics*. 2013; 59(2): 316-322.
[10] Zhang D, Li S, Sun M, O'Neill Z. An Optimal and Learning-Based Demand Response and Home Energy Management System. *IEEE Trans. on Smart Grid*. 2016; 7(4): 1790-1801.
[11] Anvari-Moghaddam A, Monsef H, Rahimi-Kian A. Optimal Smart Home Energy Management Considering Energy Saving and a Comfortable Lifestyle. *IEEE Trans. Smart Grid*. 2015; 6(1): 324-332.
[12] Wang N, Zhang J, Xia X. Energy Consumption of Air Conditioners at Differenct Temperature Set Points. *Energy and Buildings.* 2013; 65: 412-418.
[13] Ben-Tal A, Ghaoui L, Nemirovski A. S. *Robust Optimization*. Princeton, NJ, USA: Princeton Univ. Press, 2009.
[14] Jones D, Tamiz M. Practical Goal Programming, New York, NY: Springer, 2010.
[15] Rardin R. L. Optimization in Operations Research, Upper Saddle River, NJ: Pearson, 1997.